THE BIG IDEA

Rotating objects tend to keep rotating while non-rotating objects tend to remain non-rotating.

In the absence of an external force, the momentum of an object remains unchanged—conservation of momentum. In this chapter we extend the law of momentum conservation to rotation.

12.1 Rotational Inertia

✓ **The greater the rotational inertia, the more difficult it is to change the rotational speed of an object.**

12.1 Rotational Inertia

Newton’s first law, the law of inertia, applies to rotating objects.

- An object rotating about an internal axis tends to keep rotating about that axis.
- Rotating objects tend to keep rotating, while non-rotating objects tend to remain non-rotating.
- The resistance of an object to changes in its rotational motion is called **rotational inertia** (sometimes *moment of inertia*).

12.1 Rotational Inertia

Just as it takes a force to change the linear state of motion of an object, a torque is required to change the rotational state of motion of an object.

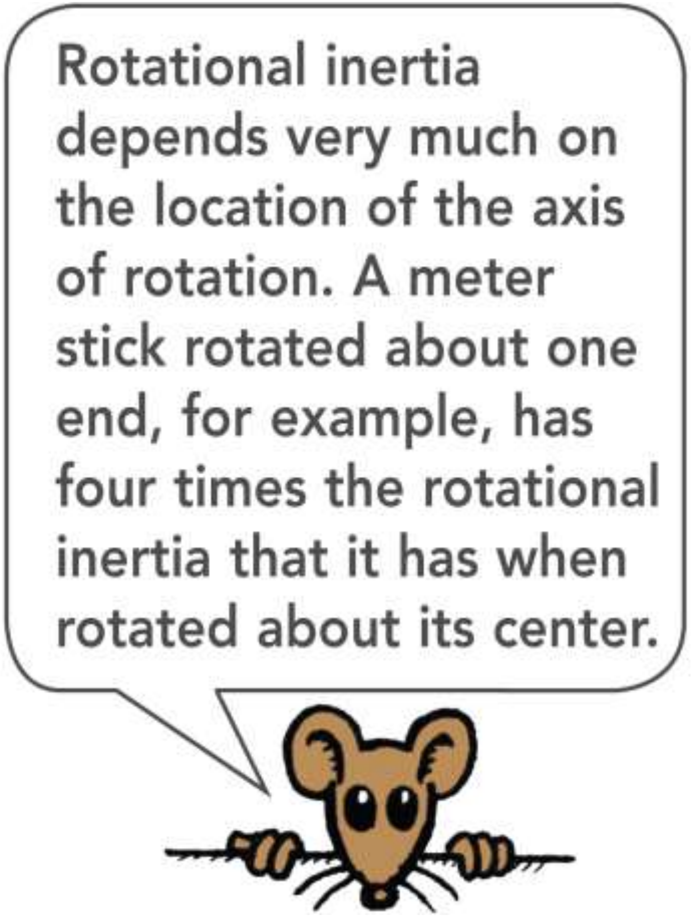
In the absence of a net torque, a rotating object keeps rotating, while a non-rotating object stays non-rotating.

12.1 Rotational Inertia

Rotational Inertia and Mass

Like inertia in the linear sense, rotational inertia depends on mass, but unlike inertia, rotational inertia depends on the *distribution* of the mass.

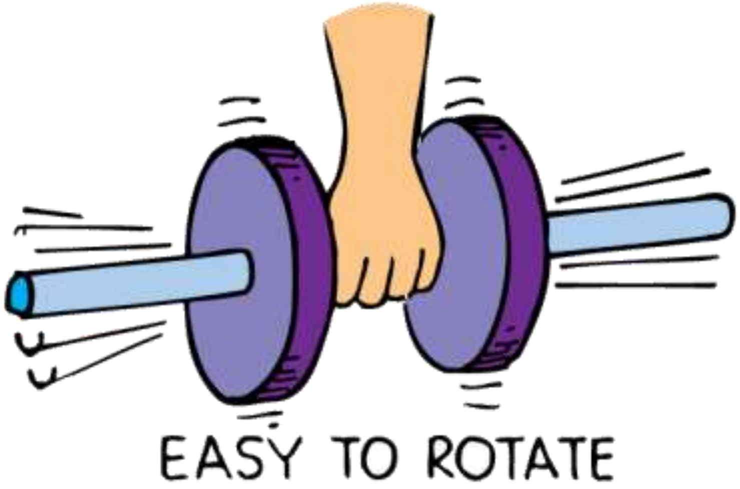
The greater the distance between an object’s mass concentration and the axis of rotation, the greater the rotational inertia.

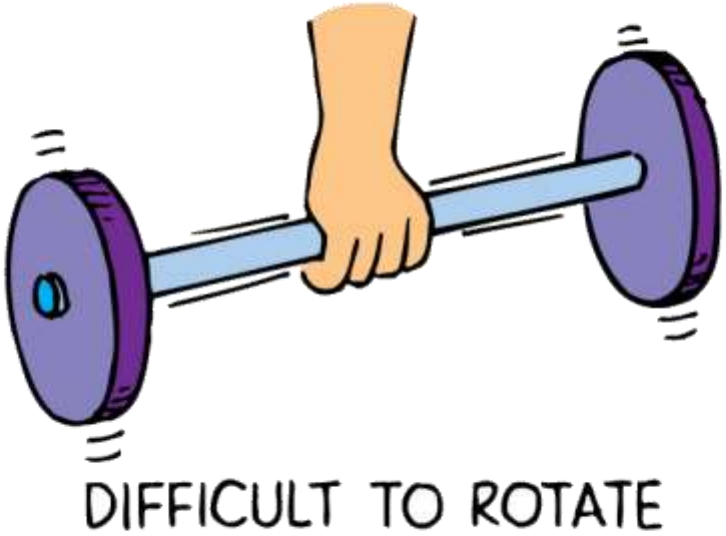


Rotational inertia depends very much on the location of the axis of rotation. A meter stick rotated about one end, for example, has four times the rotational inertia that it has when rotated about its center.

12.1 Rotational Inertia

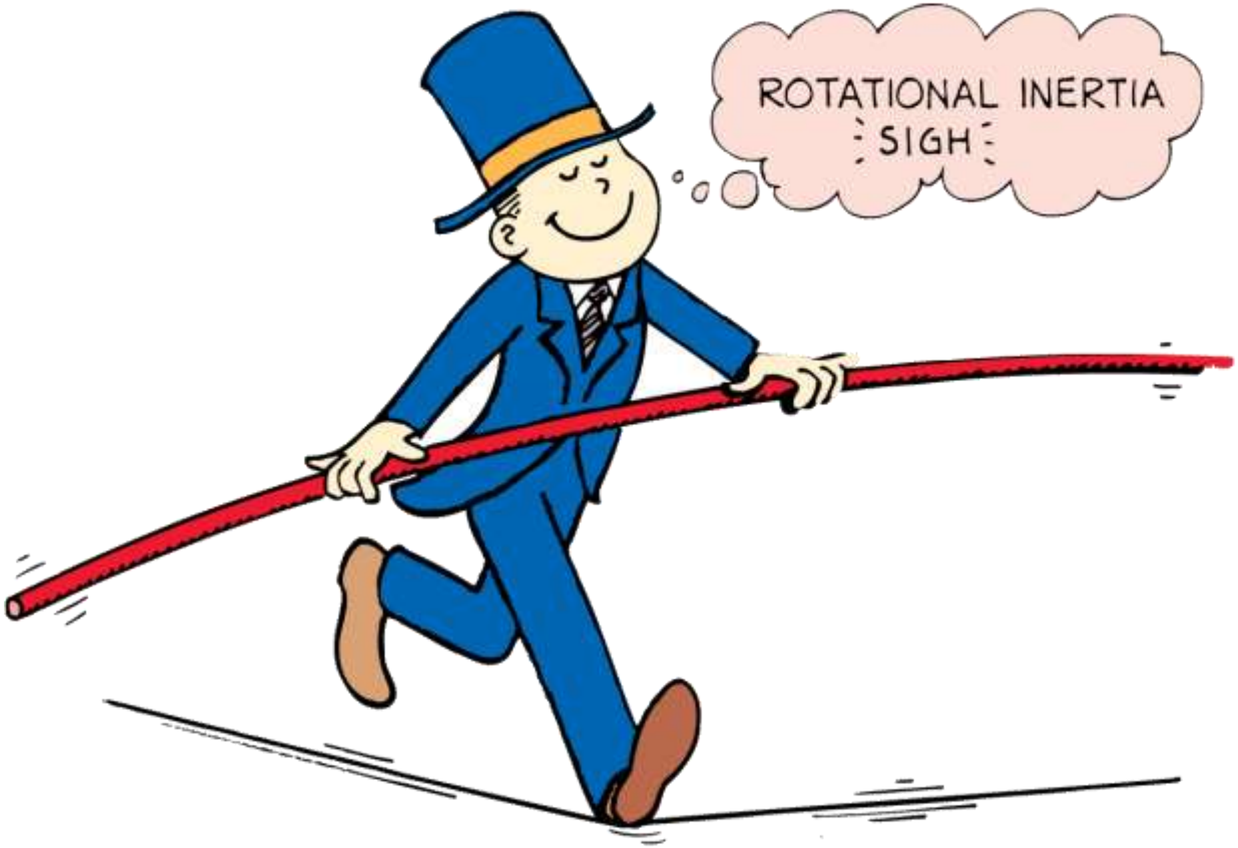
Rotational inertia depends on the distance of mass from the axis of rotation.





12.1 Rotational Inertia

By holding a long pole, the tightrope walker increases his rotational inertia.



12.1 Rotational Inertia

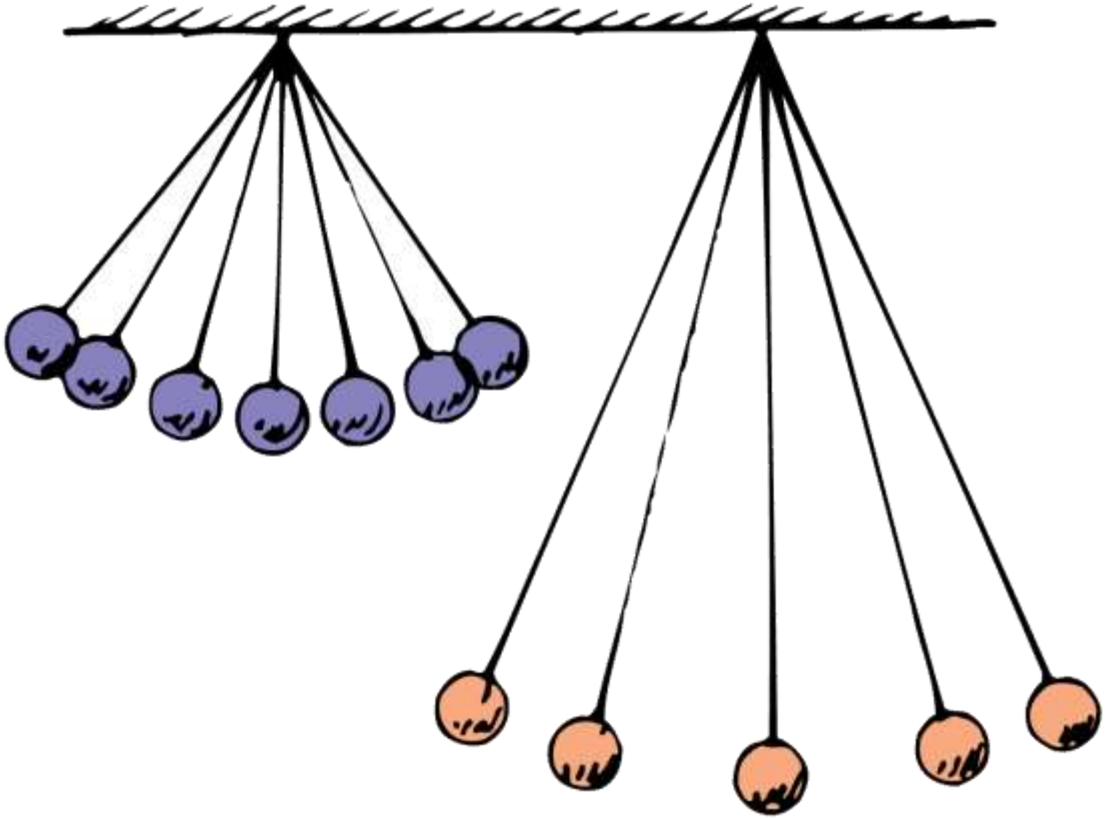
A long baseball bat held near its thinner end has more rotational inertia than a short bat of the same mass.

- Once moving, it has a greater tendency to keep moving, but it is harder to bring it up to speed.
- Baseball players sometimes “choke up” on a bat to reduce its rotational inertia, which makes it easier to bring up to speed.

A bat held at its end, or a long bat, doesn’t swing as readily.

12.1 Rotational Inertia

The short pendulum will swing back and forth more frequently than the long pendulum.



12.1 Rotational Inertia

For similar mass distributions, short legs have less rotational inertia than long legs.

12.1 Rotational Inertia

The rotational inertia of an object is not necessarily a fixed quantity.

It is greater when the mass within the object is extended from the axis of rotation.

12.1 Rotational Inertia

You bend your legs when you run to reduce their rotational inertia. Bent legs are easier to swing back and forth.

12.1 Rotational Inertia

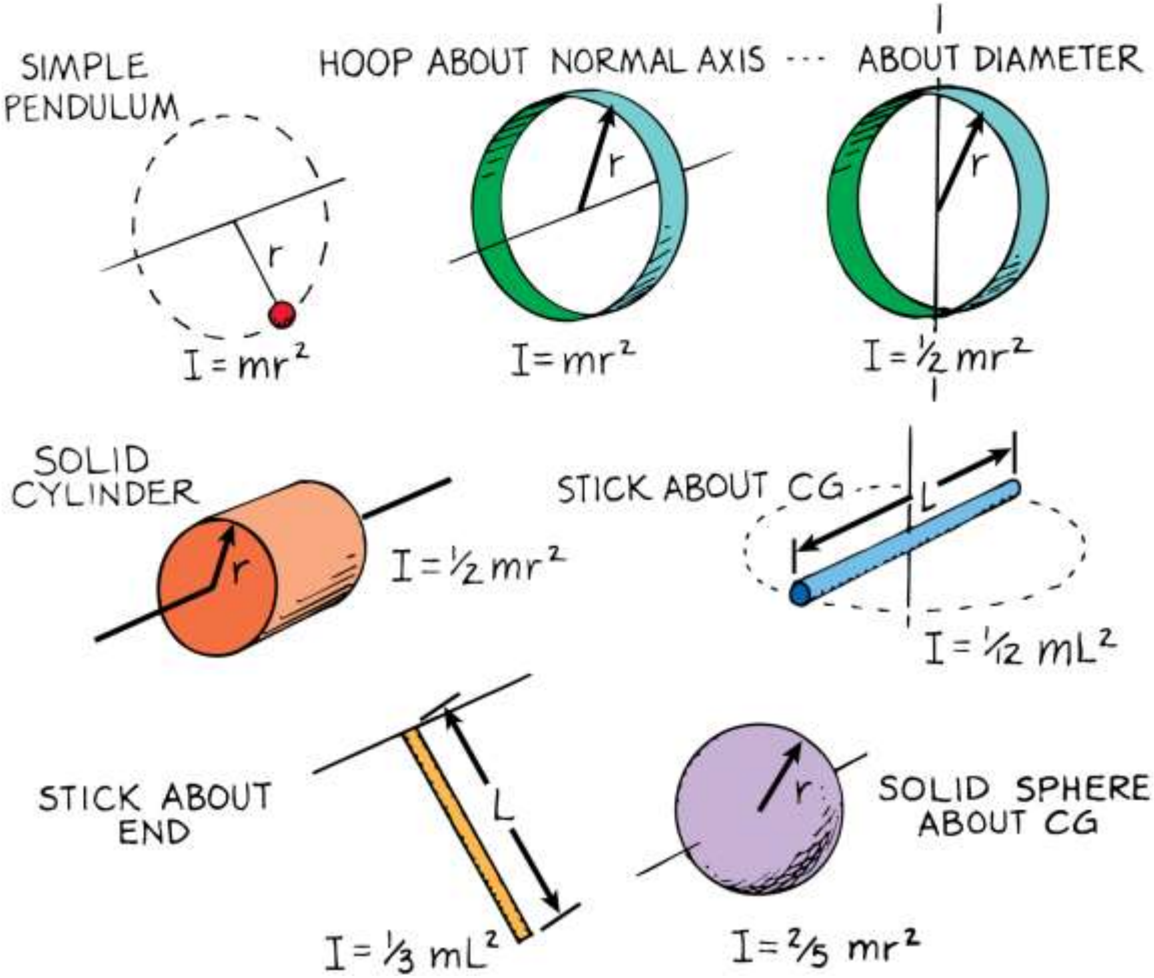
Formulas for Rotational Inertia

When all the mass m of an object is concentrated at the same distance r from a rotational axis, then the rotational inertia is $I = mr^2$.

When the mass is more spread out, the rotational inertia is less and the formula is different.

12.1 Rotational Inertia

Rotational inertias of various objects are different. (It is not important for you to learn these values, but you can see how they vary with the shape and axis.)



12.1 Rotational Inertia

think!

When swinging your leg from your hip, why is the rotational inertia of the leg less when it is bent?

12.1 Rotational Inertia

think!

When swinging your leg from your hip, why is the rotational inertia of the leg less when it is bent?

Answer:

The rotational inertia of any object is less when its mass is concentrated closer to the axis of rotation. Can you see that a bent leg satisfies this requirement?

12.1 Rotational Inertia

CONCEPT CHECK

How does rotational inertia affect how easily the rotational speed of an object changes?

12.2 Rotational Inertia and Gymnastics

✓ **The three principal axes of rotation in the human body are the longitudinal axis, the transverse axis, and the medial axis.**

12.2 Rotational Inertia and Gymnastics

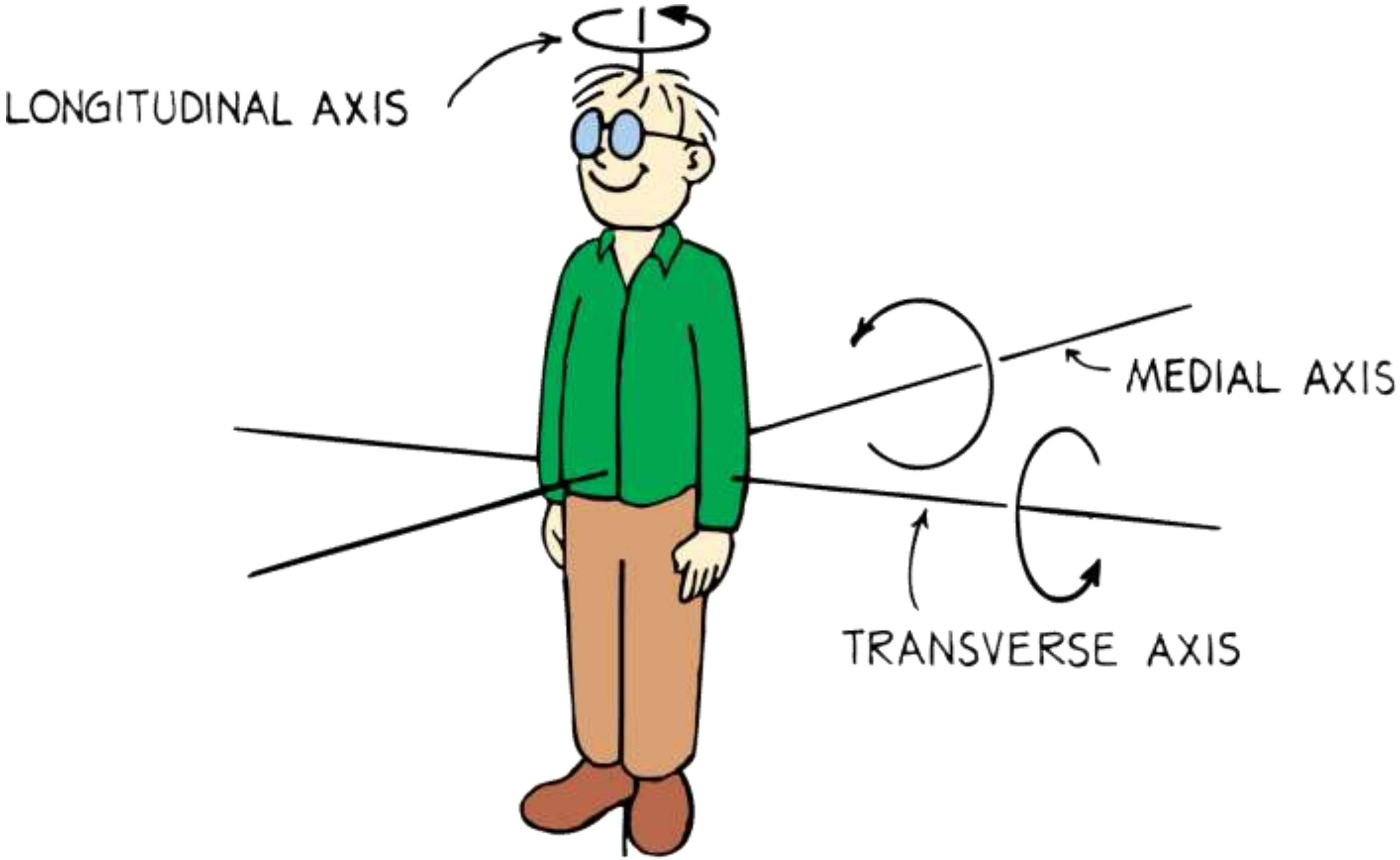
The human body can rotate freely about three principal axes of rotation.

Each of these axes is at right angles to the others and passes through the center of gravity.

The rotational inertia of the body differs about each axis.

12.2 Rotational Inertia and Gymnastics

The human body has three principal axes of rotation.



12.2 Rotational Inertia and Gymnastics

Longitudinal Axis

Rotational inertia is least about the *longitudinal axis*, which is the vertical head-to-toe axis, because most of the mass is concentrated along this axis.

- A rotation of your body about your longitudinal axis is the easiest rotation to perform.
- Rotational inertia is increased by simply extending a leg or the arms.

12.2 Rotational Inertia and Gymnastics

An ice skater rotates around her longitudinal axis when going into a spin.

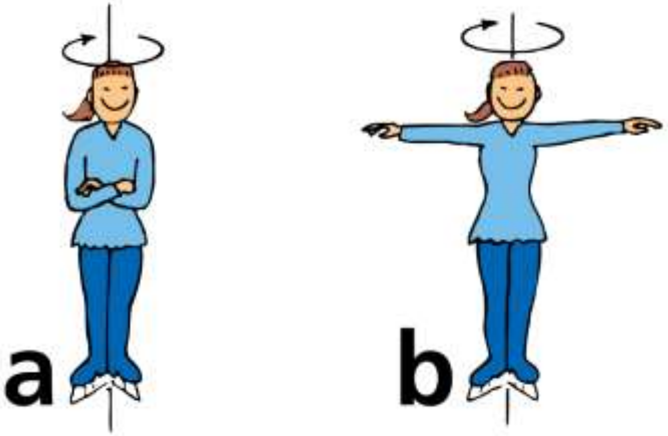
a. The skater has the least amount of rotational inertia when her arms are tucked in.

12.2 Rotational Inertia and Gymnastics

An ice skater rotates around her longitudinal axis when going into a spin.

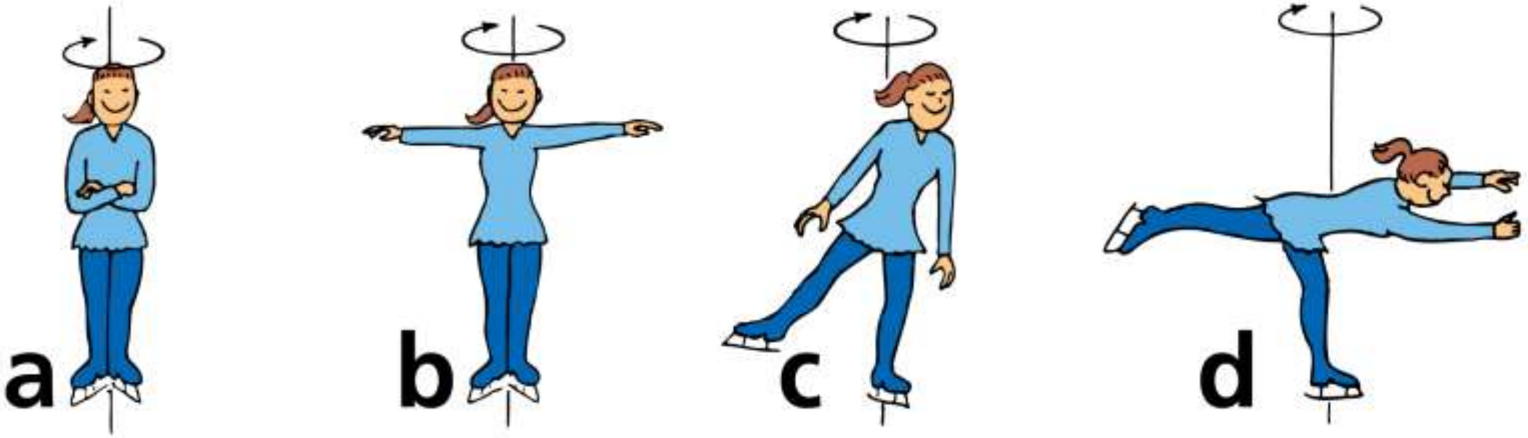
a. The skater has the least amount of rotational inertia when her arms are tucked in.

b. The rotational inertia when both arms are extended is about three times more than in the tucked position.



12.2 Rotational Inertia and Gymnastics

c and d. With your leg and arms extended, you can vary your spin rate by as much as six times.



12.2 Rotational Inertia and Gymnastics

Transverse Axis

You rotate about your *transverse axis* when you perform a somersault or a flip.

12.2 Rotational Inertia and Gymnastics

A flip involves rotation about the transverse axis.

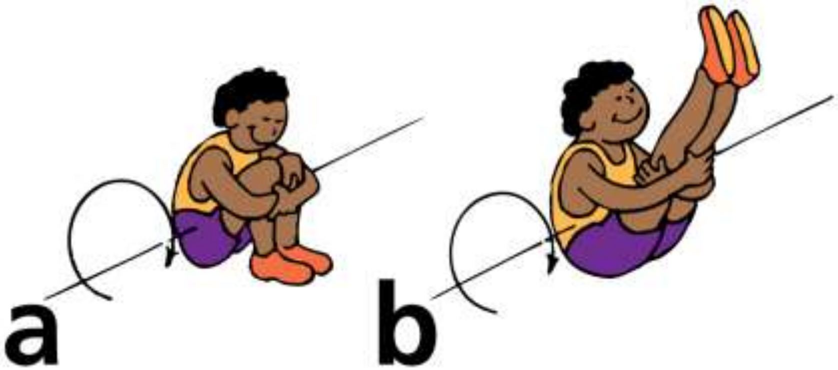
a. Rotational inertia is least in the tuck position.

12.2 Rotational Inertia and Gymnastics

A flip involves rotation about the transverse axis.

a. Rotational inertia is least in the tuck position.

b. Rotational inertia is 1.5 times greater.



12.2 Rotational Inertia and Gymnastics

A flip involves rotation about the transverse axis.

a. Rotational inertia is least in the tuck position.

b. Rotational inertia is 1.5 times greater.

c. Rotational inertia is 3 times greater.

12.2 Rotational Inertia and Gymnastics

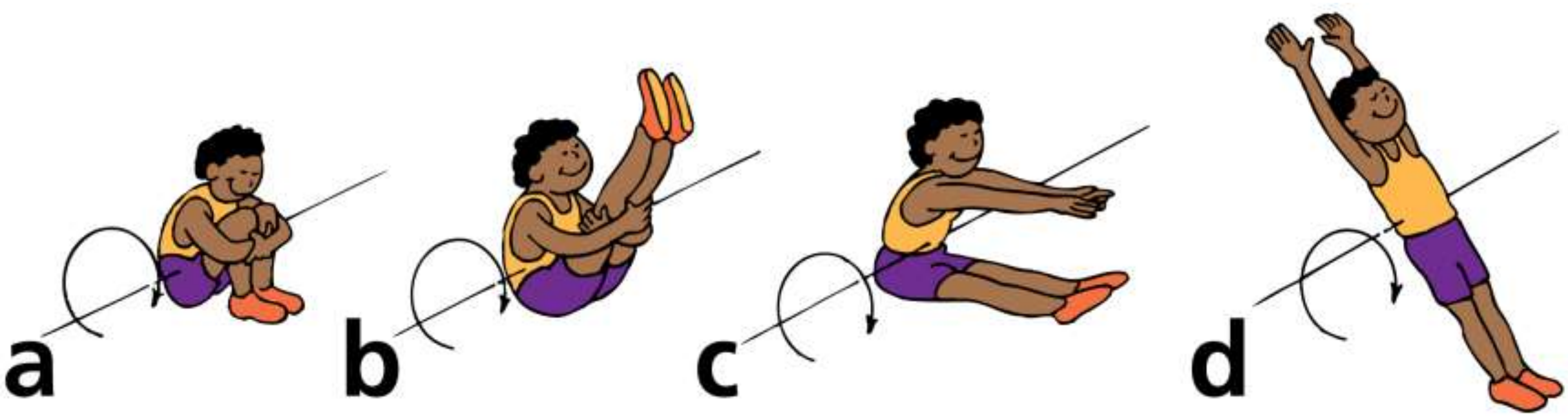
A flip involves rotation about the transverse axis.

a. Rotational inertia is least in the tuck position.

b. Rotational inertia is 1.5 times greater.

c. Rotational inertia is 3 times greater.

d. Rotational inertia is 5 times greater than in the tuck position.



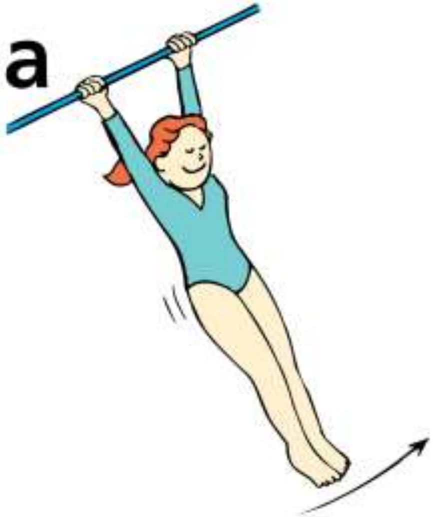
12.2 Rotational Inertia and Gymnastics

Rotational inertia is greater when the axis is through the hands, such as when doing a somersault on the floor or swinging from a horizontal bar with your body fully extended.

12.2 Rotational Inertia and Gymnastics

The rotational inertia of a body is with respect to the rotational axis.

a. The gymnast has the greatest rotational inertia when she pivots about the bar.

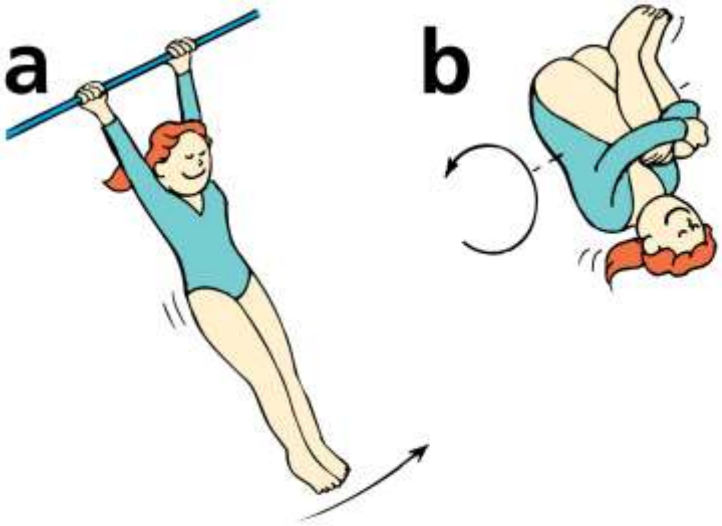


12.2 Rotational Inertia and Gymnastics

The rotational inertia of a body is with respect to the rotational axis.

a. The gymnast has the greatest rotational inertia when she pivots about the bar.

b. The axis of rotation changes from the bar to a line through her center of gravity when she somersaults in the tuck position.



12.2 Rotational Inertia and Gymnastics

The rotational inertia of a gymnast is up to 20 times greater when she is swinging in a fully extended position from a horizontal bar than after dismount when she somersaults in the tuck position.

Rotation transfers from one axis to another, from the bar to a line through her center of gravity, and she automatically increases her rate of rotation by up to 20 times.

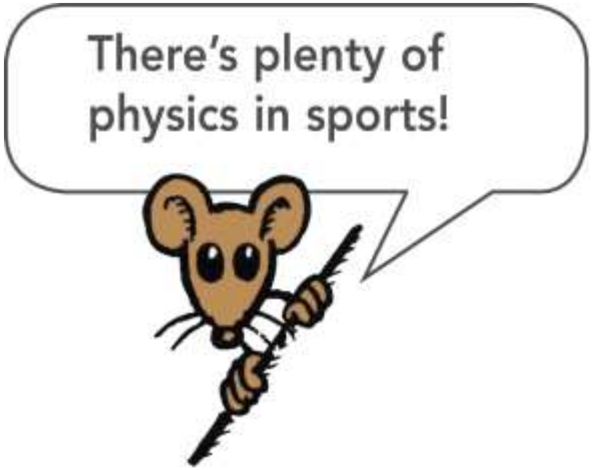
This is how she is able to complete two or three somersaults before contact with the ground.

12.2 Rotational Inertia and Gymnastics

Medial Axis

The third axis of rotation for the human body is the front-to-back axis, or *medial axis*.

This is a less common axis of rotation and is used in executing a cartwheel.



12.2 Rotational Inertia and Gymnastics

CONCEPT CHECK

What are the three principal axes of rotation in the human body?

12.3 Rotational Inertia and Rolling

✓ Objects of the same shape but different sizes accelerate equally when rolled down an incline.

12.3 Rotational Inertia and Rolling

Which will roll down an incline with greater acceleration, a hollow cylinder or a solid cylinder of the same mass and radius?

The answer is the cylinder with the smaller rotational inertia because the cylinder with the greater rotational inertia requires more time to get rolling.

12.3 Rotational Inertia and Rolling

Inertia of any kind is a measure of “laziness.”

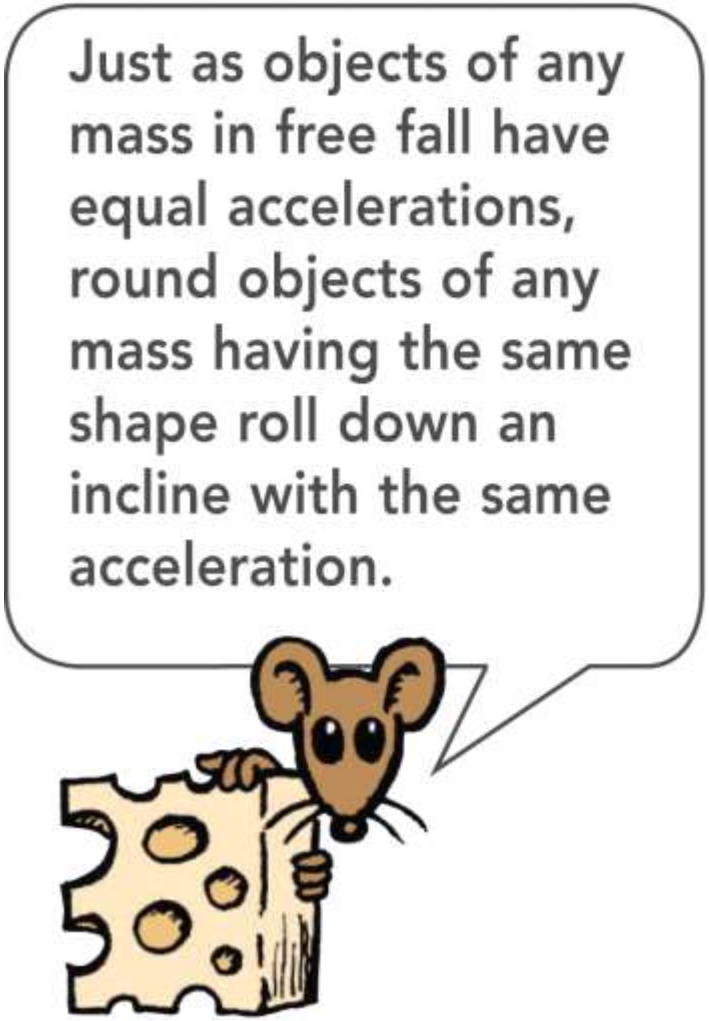
The cylinder with its mass concentrated farthest from the axis of rotation—the hollow cylinder—has the greater rotational inertia.

The solid cylinder will roll with greater acceleration.

12.3 Rotational Inertia and Rolling

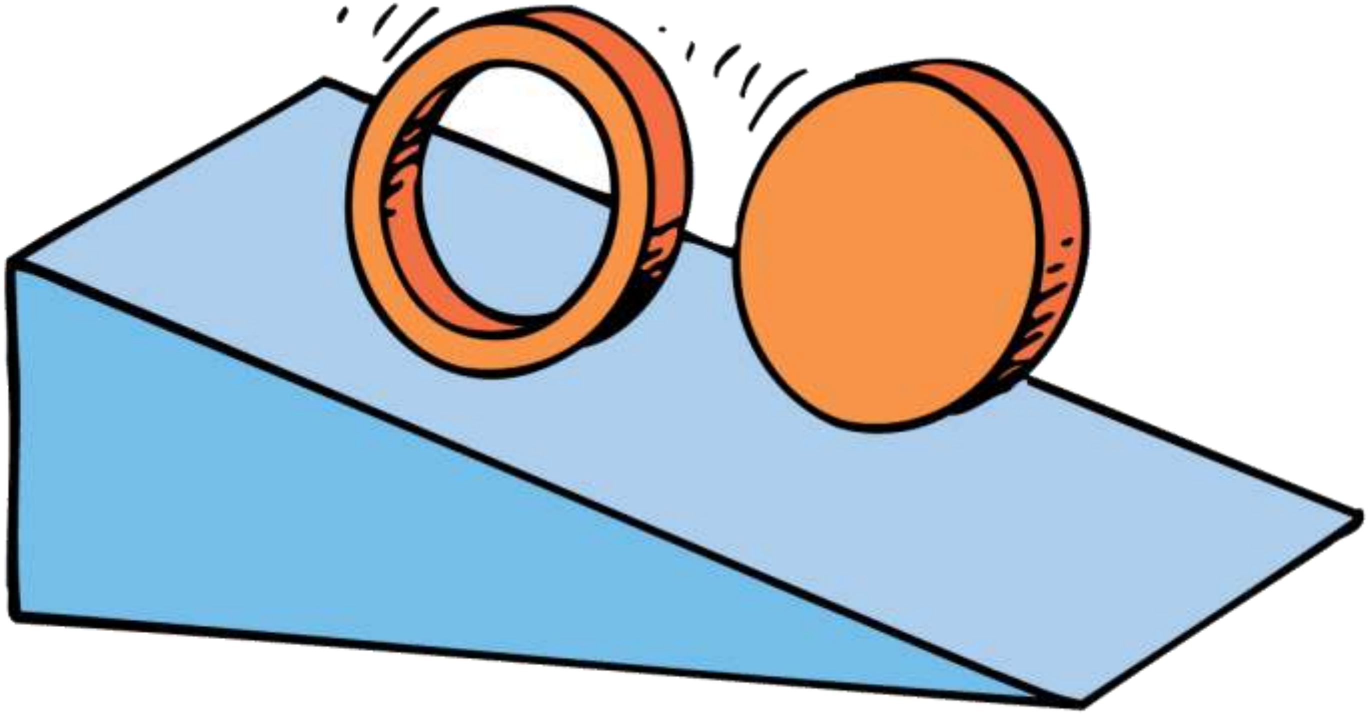
Any solid cylinder will roll down an incline with more acceleration than any hollow cylinder, regardless of mass or radius.

A hollow cylinder has more “laziness per mass” than a solid cylinder.



12.3 Rotational Inertia and Rolling

A solid cylinder rolls down an incline faster than a hollow one, whether or not they have the same mass or diameter.



12.3 Rotational Inertia and Rolling

think!

A heavy iron cylinder and a light wooden cylinder, similar in shape, roll down an incline. Which will have more acceleration?

12.3 Rotational Inertia and Rolling

think!

A heavy iron cylinder and a light wooden cylinder, similar in shape, roll down an incline. Which will have more acceleration?

Answer:

The cylinders have different masses, but the *same rotational inertia per mass,* so both will accelerate equally down the incline. Their different masses make no difference, just as the acceleration of free fall is not affected by different masses. All objects of the same shape have the same “laziness per mass” ratio.

12.3 Rotational Inertia and Rolling

think!

Would you expect the rotational inertia of a hollow sphere about its center to be greater or less than the rotational inertia of a solid sphere? Defend your answer.

12.3 Rotational Inertia and Rolling

think!

Would you expect the rotational inertia of a hollow sphere about its center to be greater or less than the rotational inertia of a solid sphere? Defend your answer.

Answer:

Greater. Just as the value for a hoop’s rotational inertia is greater than a solid cylinder’s, the rotational inertia of a hollow sphere would be greater than that of a same-mass solid sphere for the same reason: the mass of the hollow sphere is farther from the center.

12.3 Rotational Inertia and Rolling

CONCEPT CHECK

What happens when objects of the same shape but different sizes are rolled down an incline?

12.4 Angular Momentum

✓ **Newton's first law of inertia for rotating systems states that an object or system of objects will maintain its angular momentum unless acted upon by an unbalanced external torque.**

12.4 Angular Momentum

Anything that rotates keeps on rotating until something stops it.

Angular momentum is defined as the product of rotational inertia, I, and rotational velocity, ω.

angular momentum = rotational inertia × rotational velocity (ω)

$$= I \times \omega$$

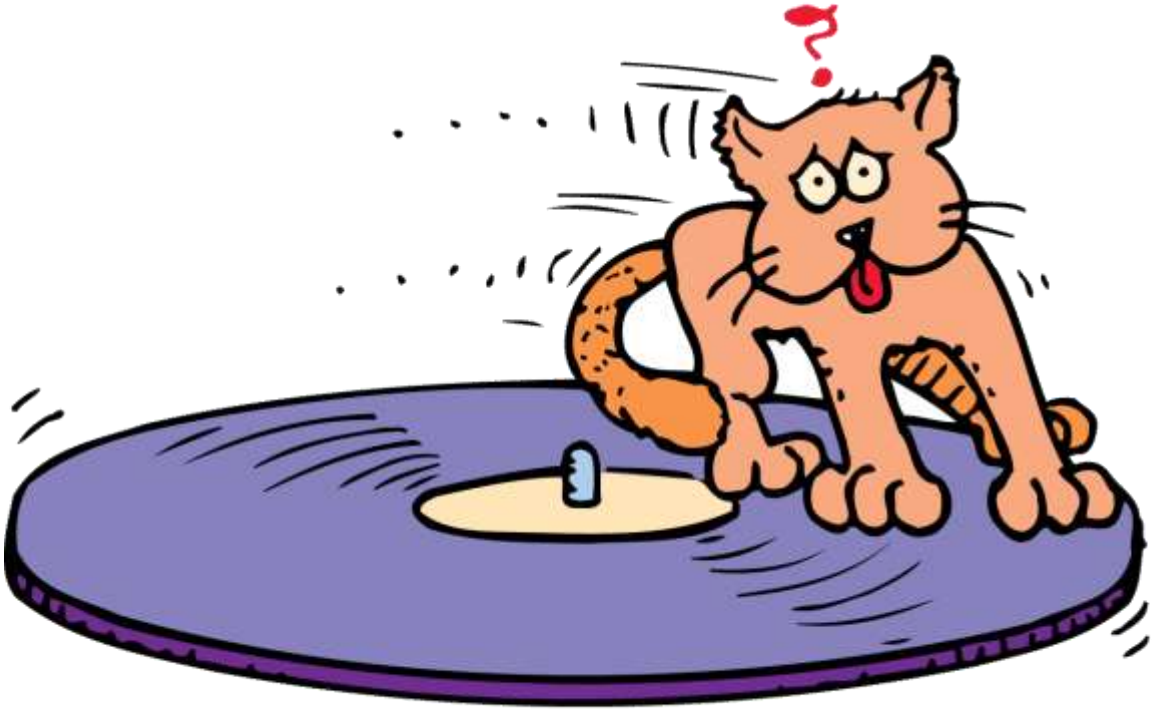
12.4 Angular Momentum

Like linear momentum, angular momentum is a vector quantity and has direction as well as magnitude.

- When a direction is assigned to rotational speed, we call it **rotational velocity.**
- Rotational velocity is a vector whose magnitude is the rotational speed.

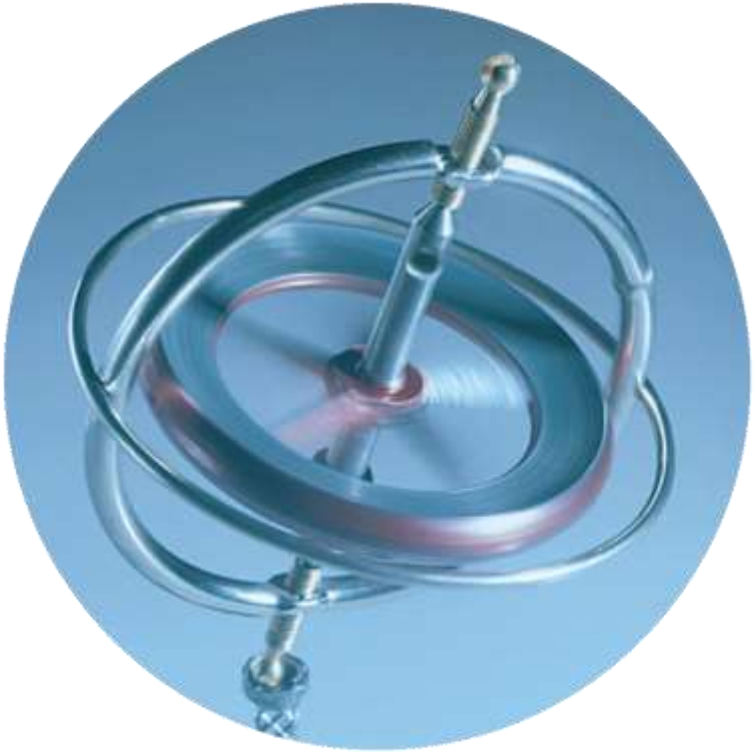
12.4 Angular Momentum

Angular momentum depends on rotational velocity and rotational inertia.



12.4 Angular Momentum

The operation of a gyroscope relies on the vector nature of angular momentum.



12.4 Angular Momentum

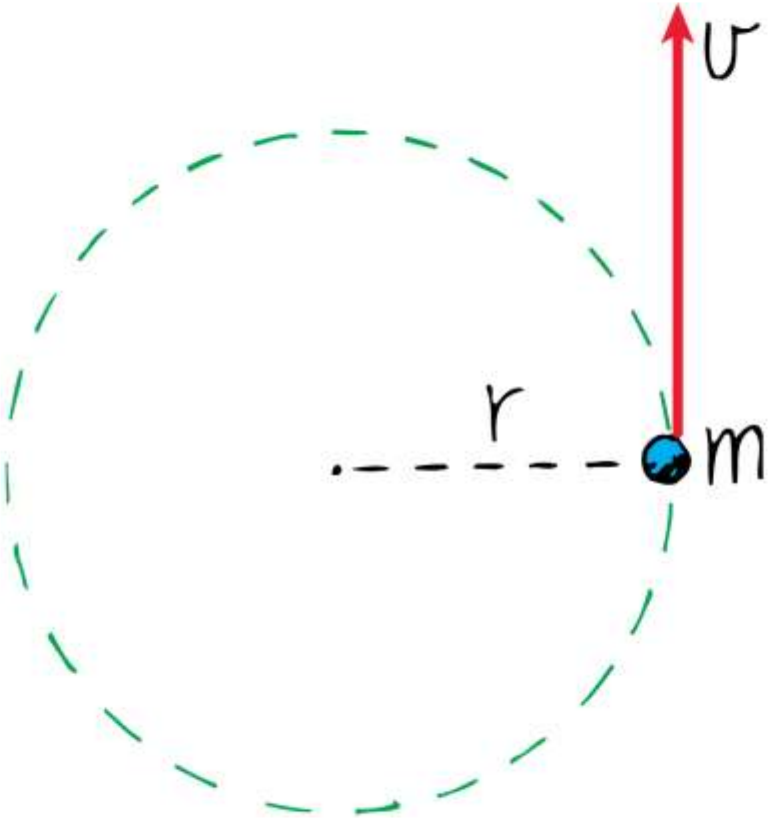
For the case of an object that is small compared with the radial distance to its axis of rotation, the angular momentum is simply equal to the magnitude of its linear momentum, mv, multiplied by the radial distance, r.

angular momentum = mvr

This applies to a tin can swinging from a long string or a planet orbiting in a circle around the sun.

12.4 Angular Momentum

An object of concentrated mass *m* whirling in a circular path of radius *r* with a speed *v* has angular momentum *mvr.*



12.4 Angular Momentum

An external net force is required to change the linear momentum of an object.

An external net torque is required to change the angular momentum of an object.

12.4 Angular Momentum

It is easier to balance on a moving bicycle than on one at rest.

- The spinning wheels have angular momentum.
- When our center of gravity is not above a point of support, a slight torque is produced.
- When the wheels are at rest, we fall over.
- When the bicycle is moving, the wheels have angular momentum, and a greater torque is required to change the direction of the angular momentum.

12.4 Angular Momentum

The lightweight wheels on racing bikes have less angular momentum than those on recreational bikes, so it takes less effort to get them turning.

12.4 Angular Momentum

CONCEPT CHECK

How does Newton’s first law apply to rotating systems?

12.5 Conservation of Angular Momentum

✓ **Angular momentum is conserved when no external torque acts on an object.**

12.5 Conservation of Angular Momentum

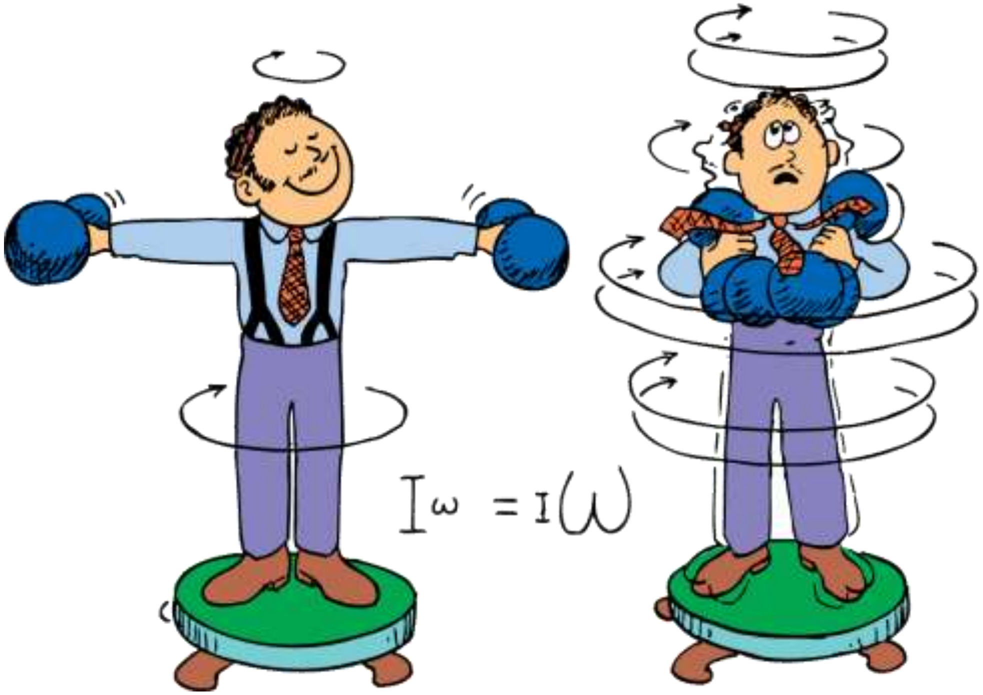
Angular momentum is conserved for systems in rotation.

The **law of conservation of angular momentum** states that if no unbalanced external torque acts on a rotating system, the angular momentum of that system is constant.

With no external torque, the product of rotational inertia and rotational velocity at one time will be the same as at any other time.

12.5 Conservation of Angular Momentum

When the man pulls his arms and the whirling weights inward, he decreases his rotational inertia, and his rotational speed correspondingly increases.



12.5 Conservation of Angular Momentum

The man stands on a low-friction turntable with weights extended.

- Because of the extended weights his overall rotational inertia is relatively large in this position.
- As he slowly turns, his angular momentum is the product of his rotational inertia and rotational velocity.
- When he pulls the weights inward, his overall rotational inertia is decreased. His rotational speed increases!
- Whenever a rotating body contracts, its rotational speed increases.

12.5 Conservation of Angular Momentum

Rotational speed is controlled by variations in the body's rotational inertia as angular momentum is conserved during a forward somersault. This is done by moving some part of the body toward or away from the axis of rotation.

12.5 Conservation of Angular Momentum

A falling cat is able to execute a twist and land upright even if it has no initial angular momentum.

During the maneuver the total angular momentum remains zero. When it is over, the cat is not turning.

This cat rotates its body through an angle, but does not create continuing rotation, which would violate angular momentum conservation.

12.5 Conservation of Angular Momentum

Although the cat is dropped upside down, it is able to rotate so it can land on its feet.

12.5 Conservation of Angular Momentum

CONCEPT CHECK

What happens to angular momentum when no external torque acts on an object?

12.6 Simulated Gravity

✓ **From within a rotating frame of reference, there seems to be an outwardly directed centrifugal force, which can simulate gravity.**

12.6 Simulated Gravity

Consider a colony of ladybugs living inside a bicycle tire.

If the wheel falls through the air, the ladybugs will be in a weightless condition and seem to float freely while the wheel is in free fall.

If the wheel is spinning, the ladybugs will feel themselves pressed to the outer part of the tire’s inner surface.

At the right spinning speed, the ladybugs will experience *simulated gravity*.

12.6 Simulated Gravity

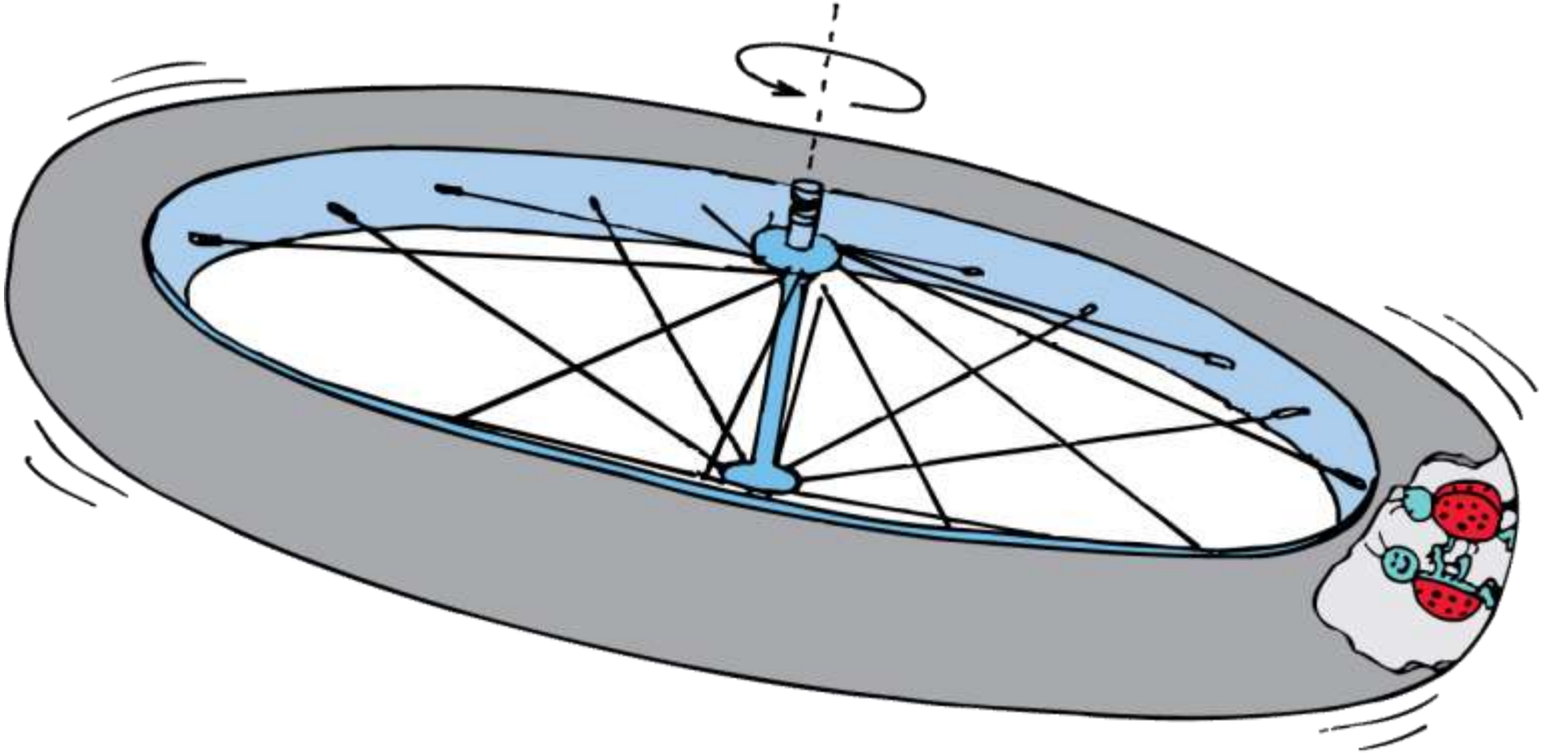
Gravity is simulated by centrifugal force.

To the ladybugs, the direction “up” is toward the center of the wheel.

The “down” direction to the ladybugs is what we call “radially outward,” away from the center of the wheel.

12.6 Simulated Gravity

If the spinning wheel freely falls, the ladybugs inside will experience a centrifugal force that feels like gravity when the wheel spins at the appropriate rate.



12.6 Simulated Gravity

Need for Simulated Gravity

Today we live on the outer surface of our spherical planet, held here by gravity.

In the future, people will likely live in huge lazily rotating space stations where simulated gravity allows them to function normally.

12.6 Simulated Gravity

Support Force

Occupants in today’s space vehicles feel weightless because they lack a support force.

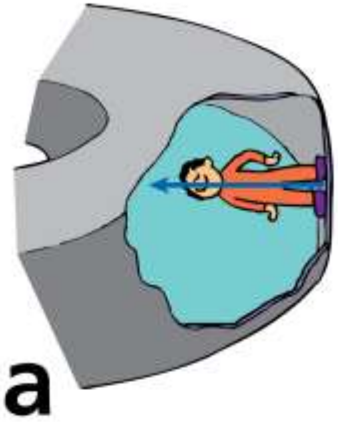
Future space travelers need not be subject to weightlessness.

Their space habitats will probably spin, effectively supplying a support force and simulating gravity.

12.6 Simulated Gravity

The man inside this rotating space habitat experiences simulated gravity.

a. As seen from the outside, the only force exerted on the man is by the floor.

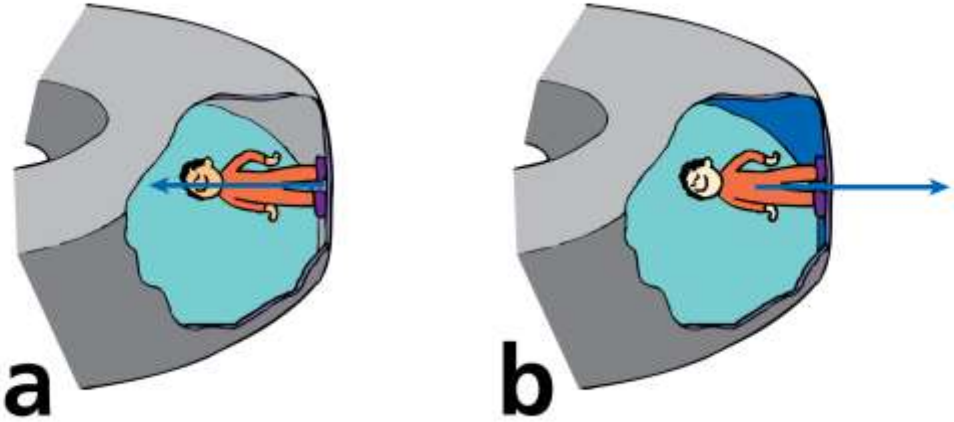


12.6 Simulated Gravity

The man inside this rotating space habitat experiences simulated gravity.

a. As seen from the outside, the only force exerted on the man is by the floor.

b. As seen from the inside, there is a fictitious centrifugal force that simulates gravity.



12.6 Simulated Gravity

As seen at rest outside the rotating system:

- The floor presses against the man (action) and the man presses back on the floor (reaction).
- The only force exerted on the man is by the floor.
- It is directed toward the center and is a centripetal force.

12.6 Simulated Gravity

As seen from inside the rotating system:

- In addition to the man-floor interaction there is a centrifugal force exerted on the man at his center of mass. It seems as real as gravity.
- Yet, unlike gravity, it has no reaction counterpart.
- Centrifugal force is not part of an interaction, but results from rotation. It is therefore called a fictitious force.

12.6 Simulated Gravity

Challenges of Simulated Gravity

The comfortable 1 *g* we experience at Earth's surface is due to gravity.

Inside a rotating spaceship the acceleration experienced is the centripetal/centrifugal acceleration due to rotation.

The magnitude of this acceleration is directly proportional to the radial distance and the square of the rotational speed.

At the axis where radial distance is zero, there is no acceleration due to rotation.

12.6 Simulated Gravity

Small-diameter structures would have to rotate at high speeds to provide a simulated gravitational acceleration of 1 *g*.

Sensitive and delicate organs in our inner ears sense rotation. Although there appears to be no difficulty at 1 RPM, many people have difficulty adjusting to rotational rates greater than 2 or 3 RPM.

To simulate normal Earth gravity at 1 RPM requires a large structure—one almost 2 km in diameter.

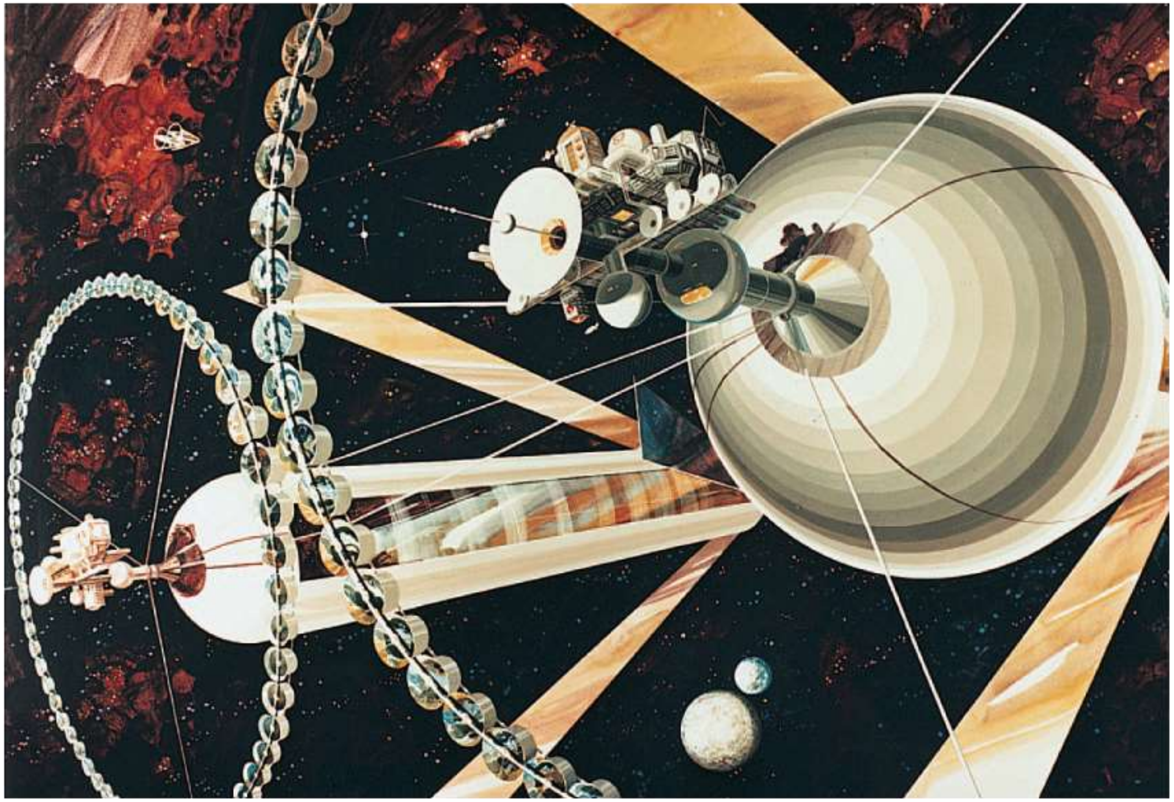
12.6 Simulated Gravity

If the structure rotates so that inhabitants on the inside of the outer edge experience 1 *g*, then halfway between the axis and the outer edge they would experience only 0.5 *g*.

The idea of a rotating space station to keep astronauts' feet on the floor, wonderfully shown in the 1968 movie *2001: A Space Odyssey*, and in Arthur C. Clarke's 1973 book *Rendezvous with Rama*, is credited to the Russian scientist Konstantin Tsiolkovsky in 1920.

12.6 Simulated Gravity

This NASA depiction of a rotational space colony may be a glimpse into the future.



12.6 Simulated Gravity

CONCEPT CHECK

How is gravity simulated?

Assessment Questions

1. The rotational inertia of an object is greater when most of the mass is located
    a. near the center.
    b. off center.
    c. on the rotational axis.
    d. away from the rotational axis.

Assessment Questions

1. The rotational inertia of an object is greater when most of the mass is located
   a. near the center.
   b. off center.
   c. on the rotational axis.
   d. away from the rotational axis.

Answer: D

Assessment Questions

2. How many principal axes of rotation are found in the human body?
   - a. one
   - b. two
   - c. three
   - d. four

Assessment Questions

2. How many principal axes of rotation are found in the human body?
   a. one
   b. two
   c. three
   d. four

Answer: C

Assessment Questions

3. For round objects rolling on an incline, the faster objects are generally those with the
   a. greatest rotational inertia compared with mass.
   b. lowest rotational inertia compared with mass.
   c. most streamlining.
   d. highest center of gravity.

Assessment Questions

3. For round objects rolling on an incline, the faster objects are generally those with the
   a. greatest rotational inertia compared with mass.
   b. lowest rotational inertia compared with mass.
   c. most streamlining.
   d. highest center of gravity.

Answer: B

Assessment Questions

4. For an object traveling in a circular path, its angular momentum doubles when its linear speed
   a. doubles and its radius remains the same.
   b. remains the same and its radius doubles.
   c. and its radius remain the same and its mass doubles.
   d. all of the above

Assessment Questions

4. For an object traveling in a circular path, its angular momentum doubles when its linear speed
   a. doubles and its radius remains the same.
   b. remains the same and its radius doubles.
   c. and its radius remain the same and its mass doubles.
   d. all of the above

Answer: D

Assessment Questions

5. The angular momentum of a system is conserved
   - a. never.
   - b. at some times.
   - c. at all times.
   - d. when angular velocity remains unchanged.

Assessment Questions

5. The angular momentum of a system is conserved
   - a. never.
   - b. at some times.
   - c. at all times.
   - d. when angular velocity remains unchanged.

Answer: B

Assessment Questions

6. Gravity can be simulated for astronauts in outer space if their habitat
   - a. is very close to Earth.
   - b. is in free fall about Earth.
   - c. rotates.
   - d. revolves about Earth.

Assessment Questions

6. Gravity can be simulated for astronauts in outer space if their habitat
   a. is very close to Earth.
   b. is in free fall about Earth.
   c. rotates.
   d. revolves about Earth.

Answer: C